QUIZ 7 SOLUTIONS: LESSONS 8-9
SEPTEMBER 18, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the general solution to $x^2y' = 2xe^{-y} - 3y'$

Solution: We need to rewrite this differential equation. Write

$$\begin{aligned} x^2y' &= 2xe^{-y} - 3y' \\ \Rightarrow \quad x^2y' + 3y' &= 2xe^{-y} \\ \Rightarrow \quad (x^2+3)y' &= 2xe^{-y} \\ \Rightarrow \quad e^y y' &= \frac{2x}{x^2+3} \\ \Rightarrow \quad e^y\, dy &= \frac{2x}{x^2+3}\, dx \end{aligned}$$

So, we see this is a separable differential equation. Because we have only y on one side and x on the other, we may integrate both sides. Hence,

$$\int e^y\, dy = \int \frac{2x}{x^2+3}\, dx.$$

The RHS is a u-sub problem. Let $u = x^2 + 3$, then $du = 2x\, dx$. Thus,

$$\begin{aligned} \int \frac{2x}{x^2-3}\, dx &= \int \frac{1}{u}\, du \\ &= \ln|u| + C \\ &= \ln|x^2+3| + C. \end{aligned}$$

Here, we do not need the absolute values because $x^2+3 > 0$ for all x. However, if we were looking at x^2-3, then we **would need** to keep the absolute values because x^2-3 does take negative values.

So,

$$\int e^y\, dy = \int \frac{2x}{x^2+3}\, dx$$

becomes

$$e^y = \ln(x^2+3) + C.$$

Solving for y, we apply ln to both sides. Therefore,

$$\boxed{y = \ln(\ln(x^2-3) + C)}.$$

We do not add absolute values on the outside, because applying ln does not mean you apply absolute values.

2. [5 pts] Find the general solution to $-t^2\dfrac{dy}{dt} - ty = t^5$ where $t > 0$

<u>Solution</u>: This is a FOLDE, but it isn't quite in the right form. Divide both sides by $-t^2$ to get

$$\frac{dy}{dt} - \frac{t}{-t^2}y = \frac{t^5}{-t^2}$$

which becomes

$$\frac{dy}{dt} + \frac{1}{t}y = -t^3.$$

Now we can apply our steps.

Step 1: Find P, Q

$P(t) = \dfrac{1}{t}, \quad Q(t) = -t^3$

Step 2: Find the integrating factor

$$\begin{aligned} u(t) &= e^{\int P(t)\,dt} \\ &= e^{\int \frac{1}{t}\,dt} \\ &= e^{\ln|t|} \\ &= |t| = t \end{aligned}$$

because we are told that $t > 0$

Step 3: Set up solution

$$\begin{aligned} yu(t) &= \int Q(t)u(t)\,dt \\ \Rightarrow \quad y \cdot \underbrace{t}_{u(t)} &= \int \underbrace{(-t^3)}_{Q(t)}\underbrace{t}_{u(t)}\,dt \\ \Rightarrow \quad y \cdot t &= \int -t^4\,dt \\ &= -\frac{1}{5}t^5 + C. \end{aligned}$$

Solving for y, we get

$$\boxed{y = -\frac{1}{5}t^4 + \frac{C}{t}}.$$